

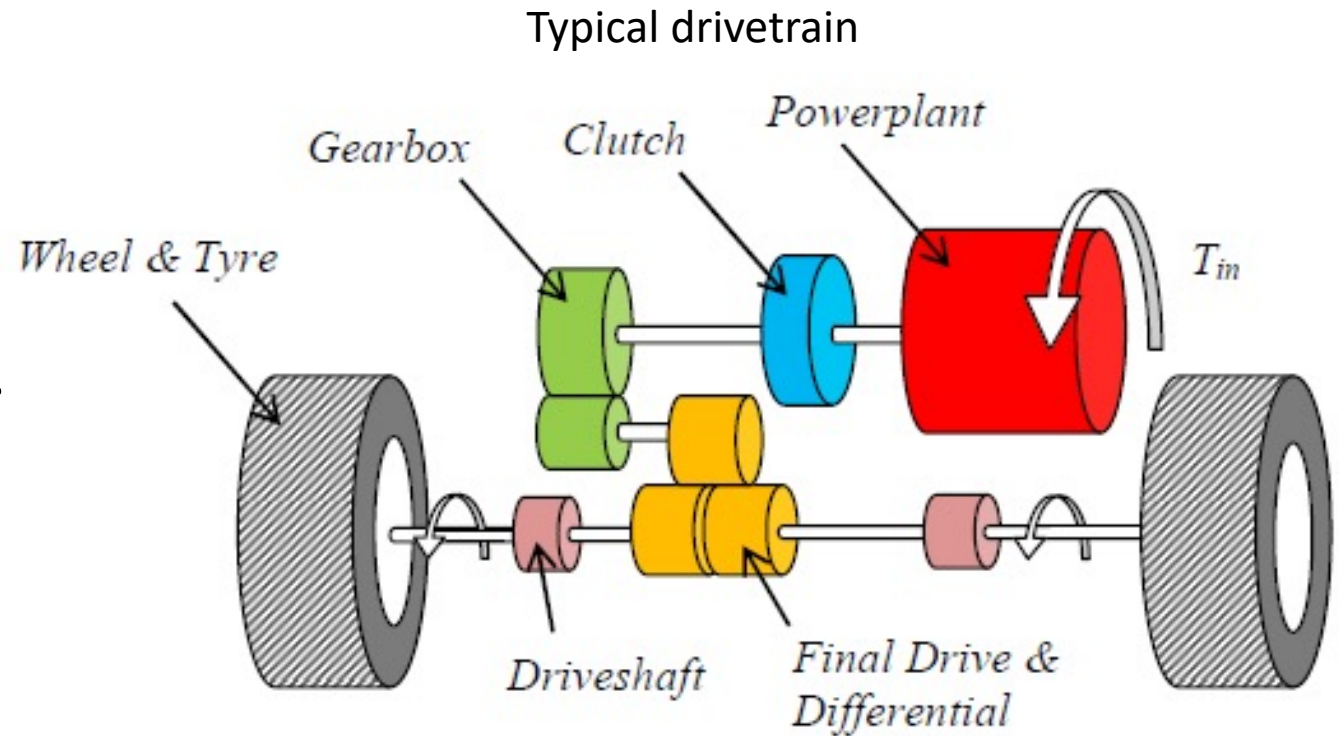
Vehicle Dynamics and Simulation

Drivetrain Modelling

Dr Byron Mason

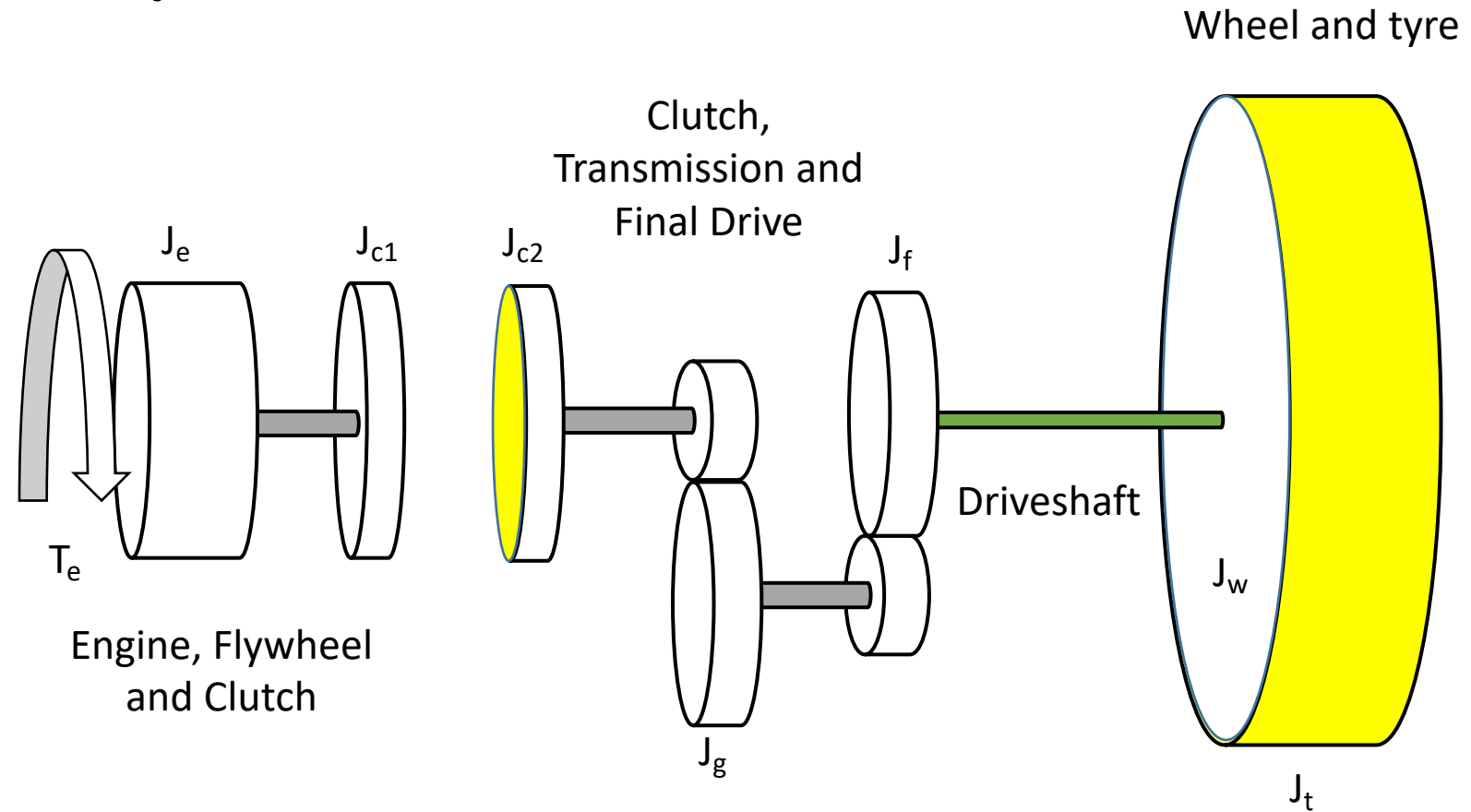
Drivetrain overview

- Drivetrain responsible for optimal delivery of torque.
 - High efficiency
 - Low NVH and driveability targets met.
- Torque introduced at source i.e. powerplant and *flows* through drivetrain for delivery at tyre / road interface.
- For our purposes a relatively simple model is required i.e. time to speed simulation and analysis
 - Drivetrain compliance and damping won't significantly affect results.



Model development

Model state 1 (3 dof)



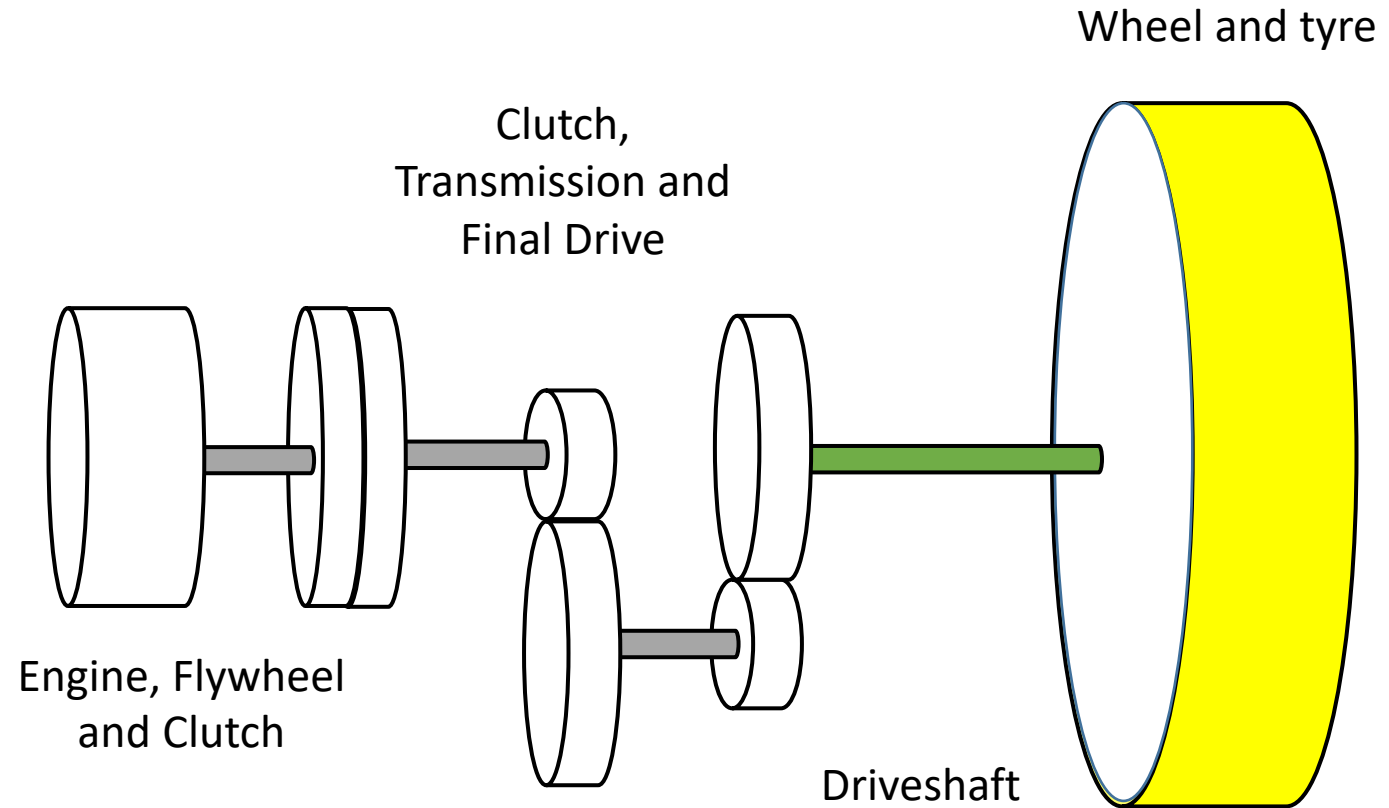
 Compliant connection (spring-damper)

 Rigid connection

 Friction interface

Model development

Model state 2 (2 dof)

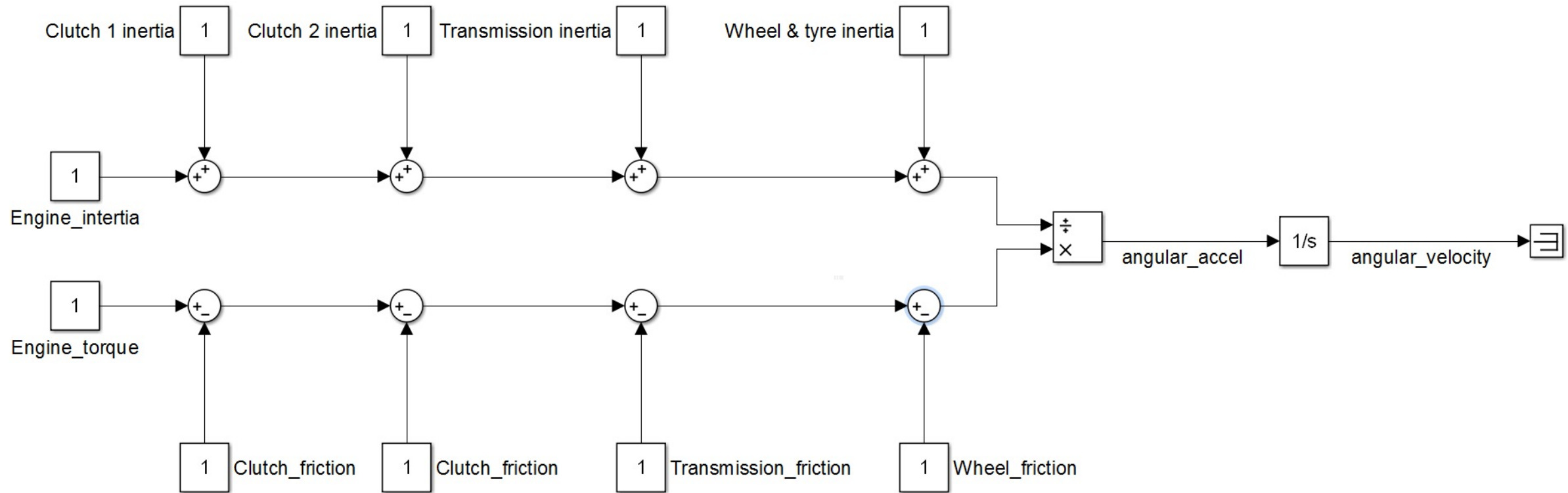


 Compliant connection (spring-damper)

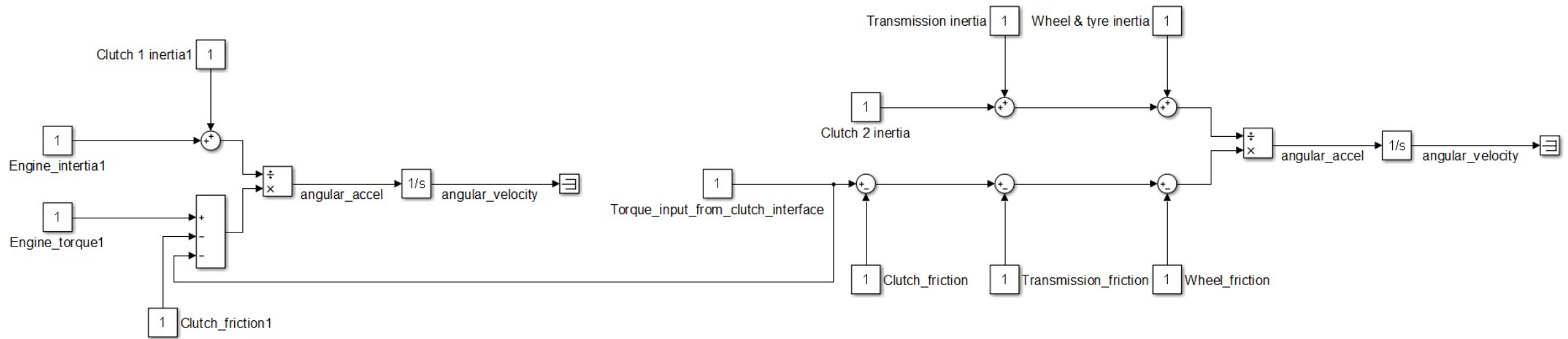
 Rigid connection

 Friction interface

1-DOF

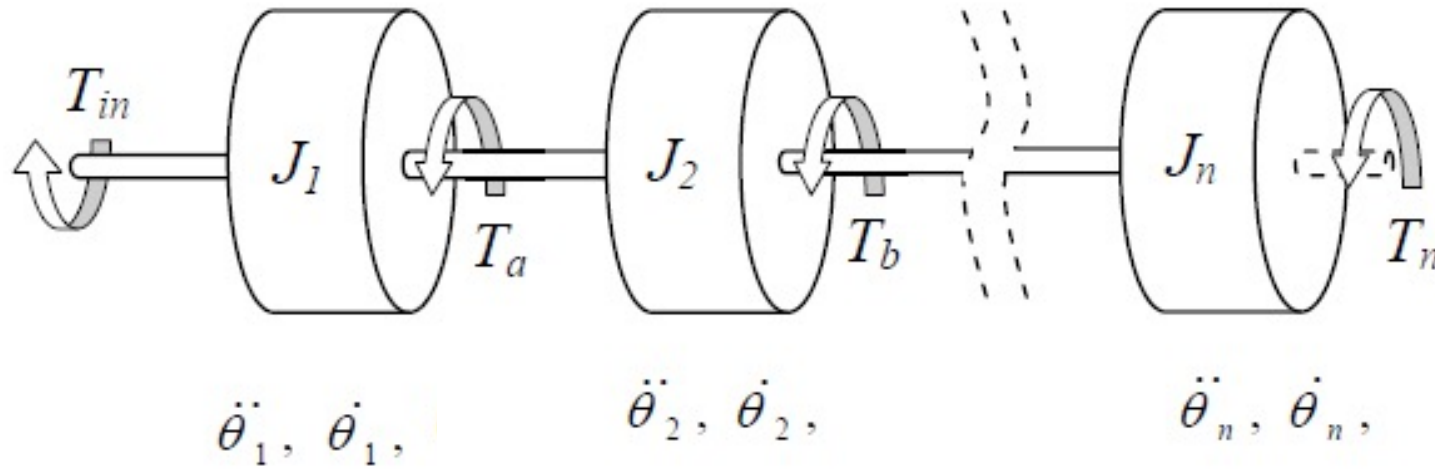


2-DOF



Model development

$$J_1 \ddot{\theta}_1 = T_{in} - T_a$$

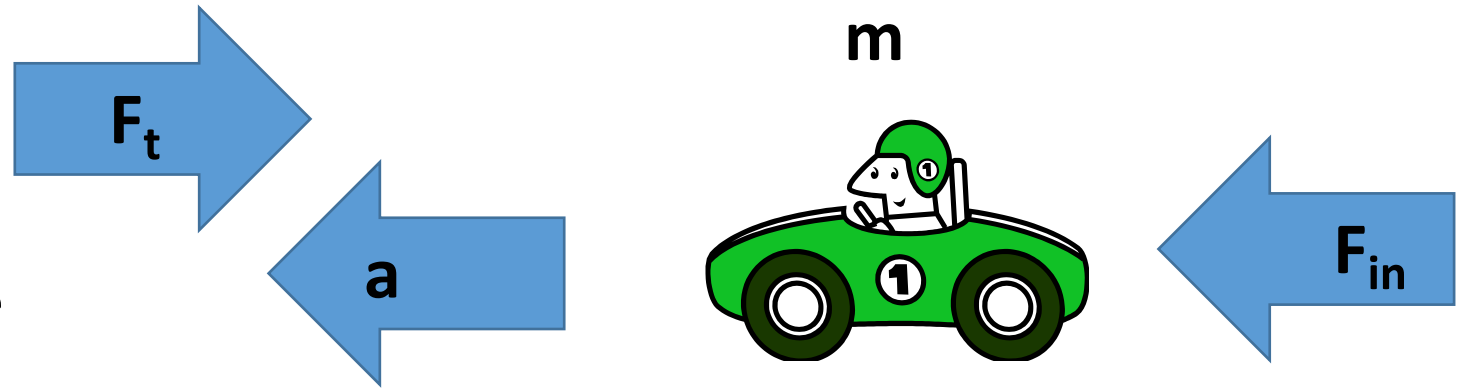


$$T_a = B(\dot{\theta}_1 - \dot{\theta}_2) + K(\theta_1 - \theta_2)$$

$$T_b = B(\dot{\theta}_2 - \dot{\theta}_3) + K(\theta_2 - \theta_3)$$

Single Inertia Vehicle Model

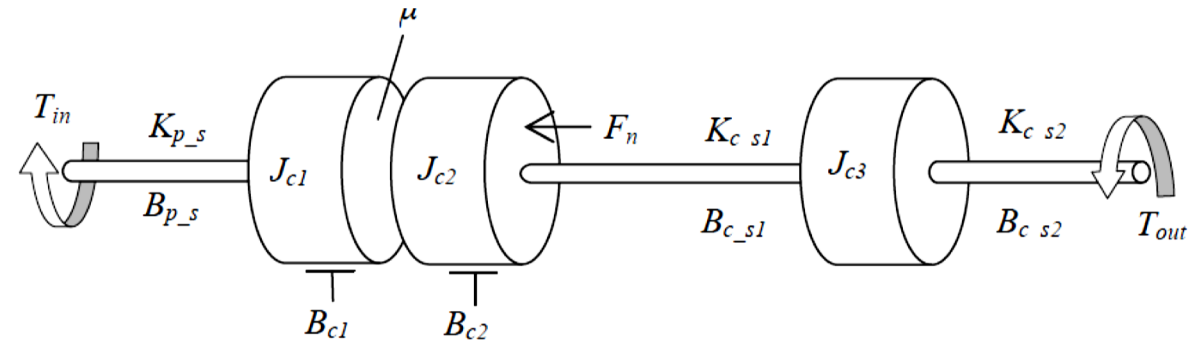
- Good representation if F_{in} is available.
- Simple to parameterise.
- Good check for validity of more complex multi-dof models.
- Disadvantages;
 - No clutch
 - Friction difficult to model
 - Unable to resolve any drivetrain performance details.



$$\ddot{x} = \frac{F}{m};$$
$$\dot{x} = \int \ddot{x} dt;$$
$$x = \int \dot{x} dt$$

Engine, Flywheel and Clutch

- Clutch is where the driveline splits and must be represented by some logic.
- Simplest model just deals with transfer of torque.
- More complex models include single or dual mass flywheels with non-linear clutch friction.
- Non-linear effects
 - Lash
 - Spring constants



Unlocked clutch $\omega_{rel} > 0$ or $T_{in} > \mu F_n$ (2 DOF)

$$J_{c1} \ddot{\theta}_{c1} = T_{in} - \mu F_n r_m$$

$$J_{c2} \ddot{\theta}_{c2} = \mu F_n r_m - T_{out}$$

Locked clutch $\omega_{rel} = 0$ or $T_{in} \leq \mu F_n$ (2 DOF)

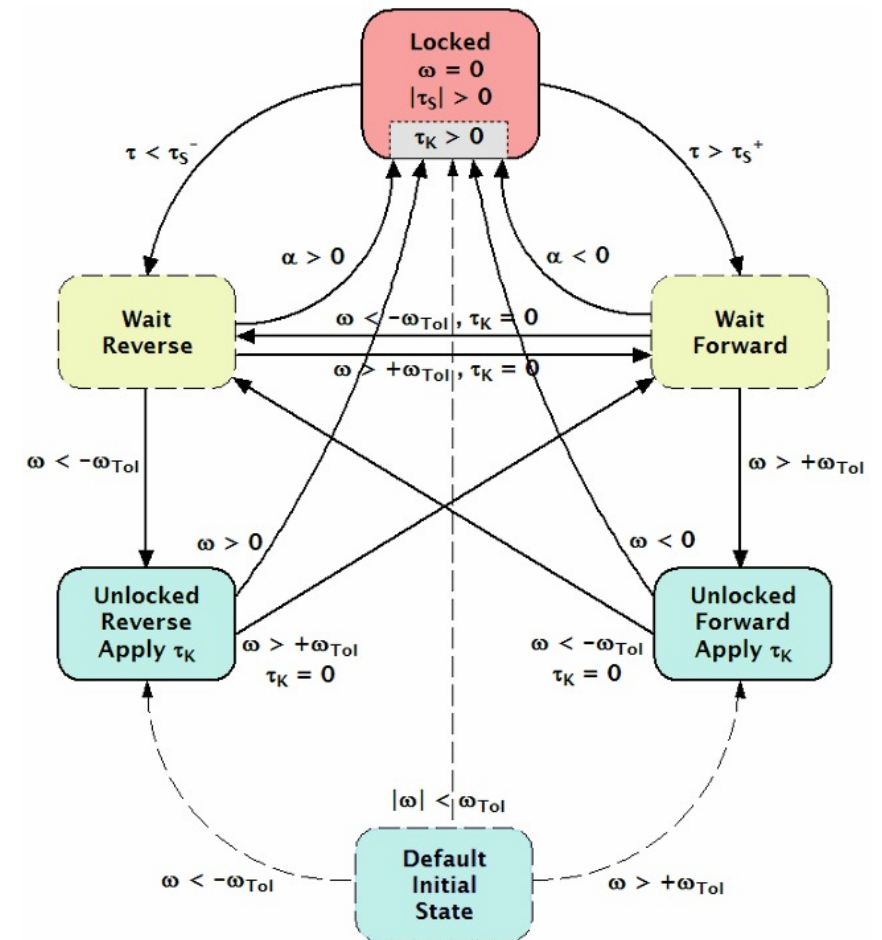
$$(J_{c1} + J_{c2}) \ddot{\theta} = T_{in} - T_{out}$$

$$\theta_{c1} = \theta_{c2}$$

Engine, Flywheel and Clutch

- Clutch switching logic can be complex due to high amplitude, high frequency oscillations that cause switching between locked and unlocked states.
- When $T_{in} > \mu N$ the clutch unlocks and then returns to locked state when $\omega_{rel} = 0$. Zero crossings i.e. oscillating directional changes also cause problems for the solver.
- Stateflow provides an easier way to implement logic.

Clutch state-diagram, typical logic

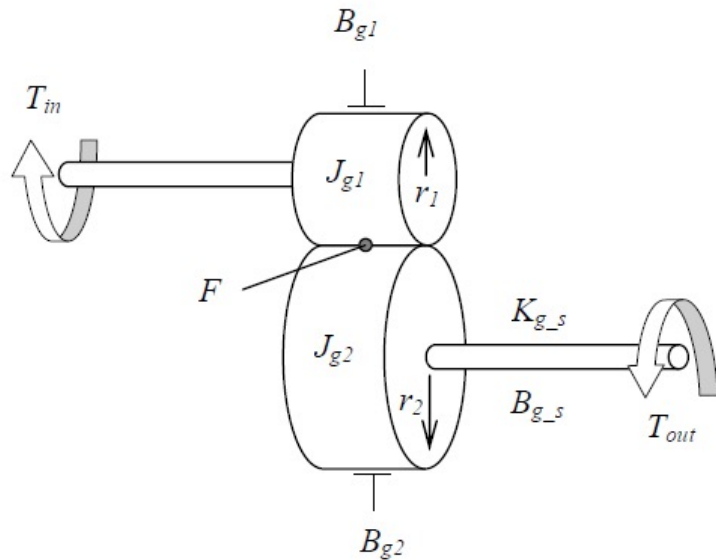


Transmission

- Conceptually very simple however speed synchronisation of input and output shafts and inertia 'switching' i.e. 1dof \rightarrow 2dof \rightarrow 1dof adds complexity.
- Lash non-linearity excites high frequency dynamics that result in very stiff system – can be dealt with by increasing damping.
- Parameterisation can also be troublesome;
 - Different frictions and inertias depending on gear ratio.



Transmission



$$J_1 \ddot{\theta}_1 = T_{in} - B_{g1} \dot{\theta}_{g1} - Fr_1$$

$$J_2 \ddot{\theta}_2 = Fr_2 - B_{g2} \dot{\theta}_{g2} - T_{out}$$

Perfect gear means that an algebraic constraint exists;

$$r_1 \theta_1 = r_2 \theta_2$$

Using the equation above, θ_1 can be eliminated (or θ_2)

$$\text{Subs for } \dot{\theta}_{g1} = \frac{r_2}{r_1} \dot{\theta}_{g2} \rightarrow J_{g1} \left(\frac{r_2}{r_1} \right) \ddot{\theta}_{g2} = T_{in} - B_{g1} \left(\frac{r_2}{r_1} \right) \dot{\theta}_{g2} - Fr_1$$

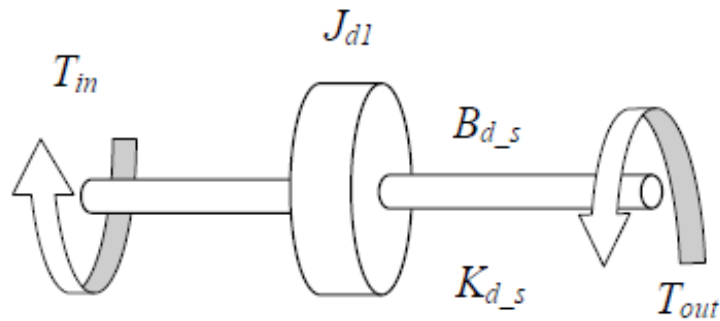
$$\times \frac{r_2}{r_1} \rightarrow J_{g1} \left(\frac{r_2}{r_1} \right)^2 \ddot{\theta}_{g2} = T_{in} \left(\frac{r_2}{r_1} \right) - B_{g1} \left(\frac{r_2}{r_1} \right)^2 \dot{\theta}_{g2} - Fr_2$$

And the complete system described (which can also be arranged for θ_{g1} ;

$$\ddot{\theta}_{g2} = \frac{T_{in} \left(\frac{r_2}{r_1} \right) - B_{g1} \left(\frac{r_2}{r_1} \right)^2 \dot{\theta}_{g2} - B_{g2} \dot{\theta}_{g2} - T_{out}}{J_{g1} \left(\frac{r_2}{r_1} \right)^2 + J_{g2}}$$

Final Drive, Differential and Driveshaft

- Driveshaft propagates torsional oscillation through compliance.
- Simple to model, less so to parameterise.



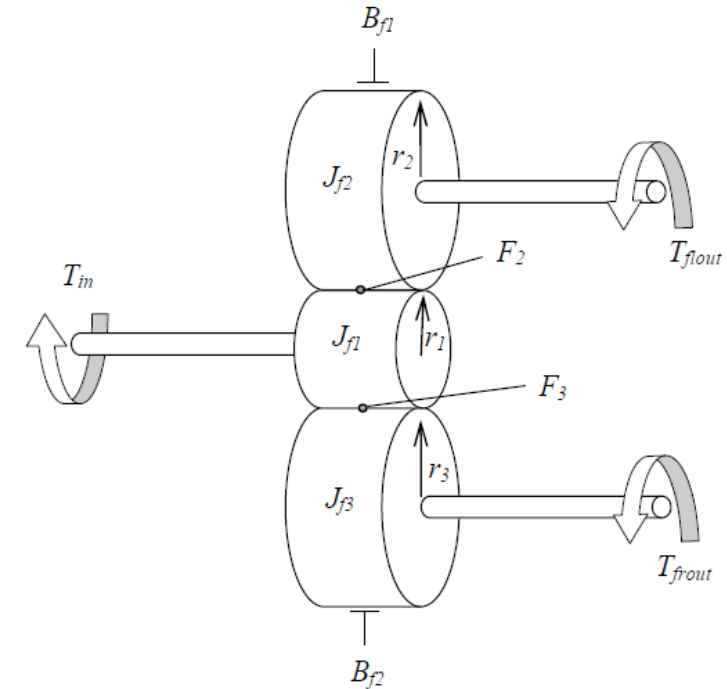
System equations

$$\ddot{\theta}_{d1} = \frac{T_{in} - T_{out}}{J_{d1}}$$

$$T_{out} = K_{d_s} (\theta_{d1} - \theta_{n+1}) + B_{d_s} (\dot{\theta}_{d1} - \dot{\theta}_{n+1})$$

Final Drive, Differential and Driveshaft

- Final drive represents final ratio change.
- Can include complex non-linearities such as lash, active/passive friction systems.
- Simple to implement at most basic level.



The three inertias are represented;

$$J_{f1}\ddot{\theta}_{f1} = T_{in} - F_2r_1 - F_3r_1$$

$$J_{f2}\ddot{\theta}_{f2} = F_2r_2 - B\dot{\theta}_{f2} - T_{f1out}$$

$$J_{f3}\ddot{\theta}_{f3} = F_3r_3 - B\dot{\theta}_{f3} - T_{f3out}$$

Subject to the constraints;

$$r_1\theta_{f1} = r_2\theta_{f2}$$

$$r_1\theta_{f1} = r_3\theta_{f3}$$

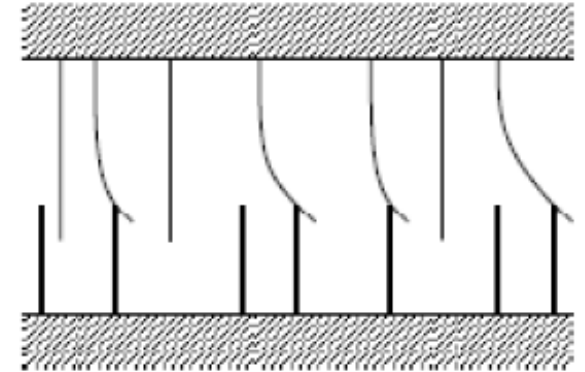
$$T_{f2out} = T_{f3out}$$

After some algebraic manipulation, to give;

$$\ddot{\theta}_{f2} = \frac{T_{in} - \left(\frac{r_1}{r_2}\right)[T_{L2} + T_{f1out}] - \left(\frac{r_1}{r_3}\right)[T_{L3} + T_{f3out}]}{\left(\frac{r_2}{r_1}\right)J_{f1} + J_{f2}\left(\frac{r_1}{r_2}\right) + J_{f3}\left(\frac{r_2}{r_3}\right)\left(\frac{r_1}{r_3}\right)}$$

Wheel and Tyres

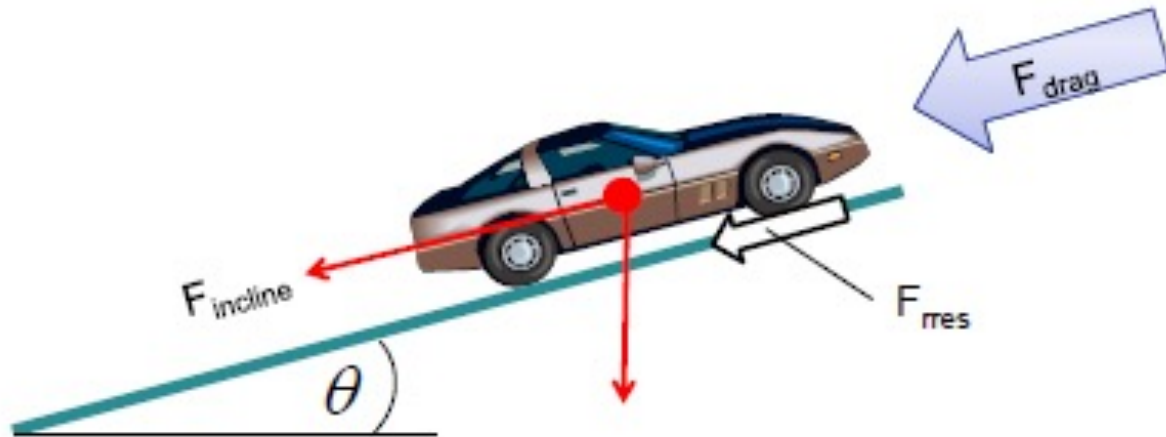
- LuGre tyre model models the tyre as the sum of solid-to-solid contact and viscous resistance of the lubricant between the tyre and road surface.
- The contribution of each is determined by the relative velocity of the tyre wrt the road.
- The LuGre tyre model remains numerically stable over the range of operation unlike some other models since as $x_v \rightarrow 0$ so instability occurs since the slip velocity $\dot{x}_{rel} \rightarrow -\infty$ i.e. $1 - \frac{r_t \theta_t}{\dot{x}_v}$



$$F_b = Kx_{b_rel} + B(\dot{x}_{t_rel})\dot{x}_{b_rel}$$

Vehicle Chassis

- Force generated in the tyre contact patch is applied to the vehicle chassis.



$$F_{rr} = mgCr$$
$$F_{inc} = mgsin\theta$$
$$F_d = \frac{1}{2}\rho AV^2 C_d$$

$$T = Fr$$